Re-Exam

Spring 2020

Important: Please make sure that you answer all questions and that you properly explain your answers. For each step write the general formula (where relevant) and explain what you do. Not only the numerical answer. If you make a calculation mistake in one of the earlier sub-questions, you can only get points for the following subquestions if the formula and the explanations are correct!

1. Consider the strategic games described below. For each of the games state how you would classify the game according to the dimensions below. If you do not have enough information to answer for a particular dimension, explain why not.

   - Are players interests aligned, totally in conflict or a mix of both?
   - Are moves sequential or simultaneous?
   - Is it a game with complete or incomplete information?
   - Is the game repeated?
   - Are mixed strategy nash equilibria possible?

   1. Rock-Paper-Scissors: On the count of three, each player makes the shape of one of the three items with her hand. Rock beats Scissors, Scissors beats Paper, and Paper beats Rock.
   2. Sealed-bid auction: Bidders on a bottle of wine seal their bids in envelopes. The highest bidder wins the item and pays the amount of his bid.
   3. Three round ultimatum bargaining game: Players alternatively make offers of how to divide 100kr. The other player can reject or accept.

Solution:

1. Rock-Paper-Scissors: On the count of three, each player makes the shape of one of the three items with her hand. Rock beats Scissors, Scissors beats Paper, and Paper beats Rock.
   - In Conflict
   - Simultaneous
   - Complete information
   - Not necessarily, but it can be.
   - Yes
2. Sealed-bid auction: Bidders on a bottle of wine seal their bids in envelopes. The highest bidder wins the item and pays the amount of his bid.
   - In Conflict
   - Simultaneous
   - Incomplete information

- No
- No

3. Three round ultimatum bargaining game. Players alternatively make offers of how to divide 100kr. The other player can reject or accept.
   - In Conflict
   - Sequential
   - Complete information (At least in the version we saw in class. Discount factor is known.)
   - No
   - No

2. A local charity is giving out hand sanitizer for free. These hand sanitizers are for people who need them and they are worried that people who can afford to buy it themselves will take advantage. Both poor students and rich professionals receive a payoff of 10 for a free hand sanitizer. The cost of standing in line is $t^2/320$ for the poor students and $t^2/160$ for the rich young professionals. t is the time measured in minutes. Assume that the charity cannot observe whether the person is a student or a young professional.

   (a) What is the minimum wait time that will achieve separation between the types?

   Solution: $t^2/160 \geq 10$
   $t = 40$

   (b) After a while the charity realized that they can identify young professionals half of the time and turn them away. That means they get no hand sanitizer and will have an embarrassment cost of 5. Does this partial identification increase or decrease the minimum wait time for young professionals?

   Solution: It will decrease the minimum waiting time that will achieve separation since the expected payoff is reduced. Half of the time young professionals will receive -5, so they are even less patient when waiting.

3. You want to decorate your house during lock down and are taking part in a Dutch Flower Auction (also called an descending first price auction). You value the flowers at 100 kr. You have observed that similar flowers have been bought for anything between 50kr and 150kr with every price in this range being equally likely.

$$b_j \sim U(50, 150).$$

   (a) Should you bid when the price is at 100kr? Why, why not? At what price, b_i, should you bid? (Hint: Define your optimal bid from before the auction starts.)

Solution:

$$\begin{aligned}\mathbb{E}[u_i(b_i, b_o^*)] &= \mathbb{P}(i \text{ wins}|b_i)(v_i - b_i) \\ &= \mathbb{P}\left(\frac{b_i - 50}{150 - 50}\right)(100 - b_i) \\ &= \left(\frac{-b_i^2 + 150b_i - 5000}{100}\right)\end{aligned}$$

Take the first order condition with respect to b_i

$$b_i^* = 75$$

If you are willing to pay 100kr you should bid when the price reaches 75kr.

(b) Given the optimal price, what is the probability that you will lose the auction?

Solution:

$$\mathbb{P}(i \text{ loses}|b_i)$$

$$= \frac{150 - b_i}{150 - 50} = \frac{150 - b_i}{100} = 0.75$$

(c) How should your strategy change if everyone had to hand in an envelope with their bid prior to the auction and would have to pay the price they stated, if they won?

Solution: You should bid the same. 75kr. The descending auction and the first-price auction are strategically equivalent.

4. Anne wants to sit outside and meet her friend Lars. She prefers to sit by the lakes, but Islands Brygge is also fine. Lars likes Islands Brygge better than the lakes. Anne doesn't know how serious Lars is about social distancing. With 60 percent chance he wants to meet her and with 40 percent chance he would rather avoid her. See the two matrixes below.

Lars - Type 1

Anne	Lakes	Islands Brygge
Lakes	$3, 1$	$-1, -1$
Islands Brygge	$-1, -1$	$1, 3$

Lars - Type 2

Anne	Lakes	Islands Brygge
Lakes	$3, -1$	$-1, 3$
Islands Brygge	$-1, 1$	$1, -1$

(a) Find the Baysian Nash Equilibria of the game.

Solution: First check for equilibria where Anne plays Lakes. In this case, Lars Type 1 plays Lakes and Type 2 plays IB. Denote this strategy by $s*_2$. The expected payoffs are

- **Anne**: Expected payoff from playing *Lakes* is

$$0,6*(3)+0,4*(-1)=1,4.$$

- Expected payoff from playing *IB* is

$$0,6*(-1)+0,4*(1)=-0,2.$$

Clearly Anne will not play IB. Thus, (Lakes, $s*_2$) is a BNE. We now check for equilibria where Anne plays IB. In this case, t1 plays IB and t2 plays Lakes. Denote this strategy by $s**_2$. The expected payoffs are

- **Player 1**: Expected payoff from playing *IB* is

$$0,6*(1)+0,4*(-1)=0,2$$

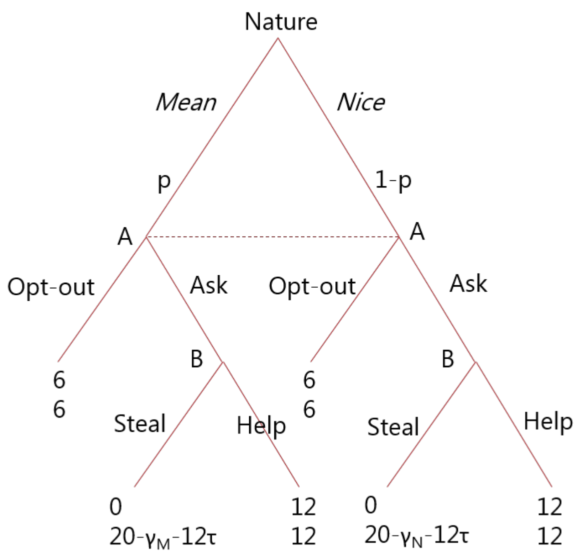
- Expected payoff from playing *Lakes* is

$$0,6*(-1)+0,4*(3)=0,6.$$

It is still better to play Lakes. Thus, (IB, $s**_2$) is not a BNE.

5. You can't go out of the house because of quarantine, so you decide to ask your neighbor to help you with grocery shopping. With 70 percent chance your neighbor is nice and with 30 percent he is mean. If your neighbor is mean, he will have a fixed utility $\gamma_M = -2$ from not buying your groceries. If he is nice, he will feel guiltier if he doesn't buy your groceries and have a fixed utility of $\gamma_N = -6$. Your neighbor has a belief about your belief of him helping you with your shopping vs just keeping the money. This is denoted with τ.

   (a) Have a look at the game below. What type of game is this? What makes it different from a traditional game?



Solution: This type of game is a psychological game. It is also a dynamic game of incomplete information. In a psychological game the payoff depends on beliefs and monetary payoff.

(b) What is the minimum belief about your belief τ that your NICE neighbor needs to have in order for him to buy you groceries rather than take the money?

Solution: For Player B, the nice neighbor:

$$\begin{aligned} U_{Steal} &\leq U_{Help} \\ 20 - 6 - 12\tau &\leq 12 \\ \tau &\geq 1/6 \end{aligned}$$

(c) What is the minimum belief about your belief τ that your MEAN neighbor needs to have in order for him to buy you groceries rather than take the money. Compare to the value of b) and explain intuitively.

Solution: For Player B, the mean neighbor:

$$\begin{aligned} U_{Steal} &\leq U_{Help} \\ 20 - 2 - 12\tau &\leq 12 \\ \tau &\geq 1/2 \end{aligned}$$

The belief of the mean neigbor about the belief of you thinking you will be helped needs to be larger, so he will feel guiltier for not helping. The nice neigbor already feels guiltier for not helping.

(d) Now assume that both types have a belief about your belief of $\tau = 1/4$. Should you ask your neighbor to help you or opt-out? Show numerically.

Solution: Given a belief of $\tau = 1/4$ the nice neighbor will help you out, while the mean neighbor will take the money. So your expected value from asking is:

$$\begin{aligned} \mathbb{E} = 0,7 * 12 + 0,3 * 0 &= 8,4 \\ 8,4 &> 6 \end{aligned}$$

Asking give you a higher expected value than opting-out.

(e) What could you do to increase the chance that either of your neighbor types will help you out?

Solution: You could thank you neighbor in advance or give him your keys to your apartment door to signal that you strongly believe that he will help you. Anything that will increase his belief about your belief of him helping you.